Large-Scale Variation of Solar Wind Electron Properties from Quasi-Thermal Noise Spectroscopy: *Ulysses* Measurements

G. Le Chat · K. Issautier · N. Meyer-Vernet · S. Hoang

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Abstract The transport of energy in space plasmas, especially in the solar wind, is far from being understood. Measuring the temperature of the electrons and their non-thermal properties is essential to understand the transport properties in collisionless plasmas. Quasithermal noise spectroscopy is a reliable tool for measuring the electron temperature accurately since it is less sensitive to the spacecraft perturbations than particle detectors. We apply this method to *Ulysses* radio data obtained during the first pole-to-pole fast latitude scan in the high-speed solar wind, using a kappa function to describe the electron velocity distribution. We deduce the variations with heliocentric distance between 1.5 and 2.3 AU in the fast solar wind at high latitude in terms of three fitting parameters: the electron density varies as $n_e \propto R^{-1.96\pm0.08}$, the electron temperature as $T_e \propto R^{-0.53\pm0.15}$, and the kappa index of the distribution remains constant at $\kappa = 2.0 \pm 0.2$. These observations agree with the predictions of the exospheric theory.

Keywords Plasma physics · Solar wind

1. Introduction

In the solar wind and other collisionless plasmas, the mechanism of energy transport remains an open question. Because of the large difference in mass between ions and electrons, electrons do transport energy whereas ions transport momentum. Consequently, the electron temperature and their non-thermal properties are of prime interest to understand the transport of energy in the solar wind.

Observations of solar wind electrons show that they cool off with a behavior intermediate between isothermal and adiabatic (see Table 1 of Maksimovic, Gary, and Skoug, 2000). Assuming that the electron temperature can be fitted with a power law of the distance (*R*) to the Sun: $T \propto R^{\beta}$, β is observed to range between 0 (isothermal) and -4/3 (adiabatic),

G. Le Chat (🖂) · K. Issautier · N. Meyer-Vernet · S. Hoang

LESIA, Observatoire de Paris, CNRS, UPMC, Université Paris Diderot, 5 Place Jules Janssen,

⁹²¹⁹⁵ Meudon, France

e-mail: gaetan.lechat@obspm.fr

with a large scatter in the measurements. For the total kinetic temperature, β is found between -0.2 and -0.9 (Montgomery, Bame, and Hundhausen, 1968; Feldman *et al.*, 1978; Marsch *et al.*, 1989; Pilipp *et al.*, 1990; Maksimovic, Gary, and Skoug, 2000), whereas for the electron core temperature, β is found between -0.3 and -1.1 (Ogilvie and Scudder, 1978; Feldman *et al.*, 1979; Sittler and Scudder, 1980; Sittler, Scudder, and Jessen, 1981; Scime *et al.*, 1994; Phillips *et al.*, 1995; Maksimovic, Hoang, and Bougeret, 1995; Issautier *et al.*, 1998). The absence of overall agreement between the observations is due to various causes:

- *i*) the observations have been carried out in different latitudinal and radial ranges and in different phases of the solar activity;
- *ii*) classification of data by the solar wind speed has not always been done;
- iii) most data have been acquired in the ecliptic where contamination by compression regions and temporal variations is difficult to eliminate;
- *iv*) many different data acquisition, reduction, and fitting techniques have been used.

In this paper, we present the application of the quasi-thermal noise spectroscopy using a kappa function (Section 2) to a selection of data obtained by the URAP (Unified RAdio and Plasma wave) experiment onboard *Ulysses* (Section 3) in order to give radial variations of the electron density, temperature and kappa index in the high-latitude fast solar wind (Section 4).

2. Quasi-Thermal Noise Spectroscopy

A passive electric antenna is sensitive to the fluctuations of the electric potential produced by the motions of the ambient electrons and ions. As long as the solar wind has no instability at frequencies around its plasma frequency, which is usually the case, this quasi-thermal noise (QTN) is completely determined by the particle velocity distributions (Rostoker, 1961) in the frame of the antenna as basically described by Meyer-Vernet and Perche (1989). A sum of two Maxwell functions has mainly been used for modeling the electron velocity distribution in quasi-thermal noise analysis (Issautier *et al.*, 1998, 2008; Issautier, Moncuquet, and Hoang, 2004). Such a velocity distribution does not adequately model the suprathermal electrons (Pierrard and Lazar, 2010). Therefore, in this paper we calculate the QTN theoretically using a kappa function for electron velocity distributions (Le Chat *et al.*, 2009). This method allows a much better accuracy for the total electron temperature measurement (Le Chat *et al.*, 2010). We use the following definition of the kappa function:

$$f_{\kappa}(v) = \frac{A}{(1 + v^2/\kappa v_0^2)^{\kappa+1}}$$
(1)

with:

$$A = \frac{\Gamma(\kappa + 1)}{(\pi \kappa)^{3/2} v_0^3 \Gamma(\kappa - 1/2)},$$
(2)

where $\Gamma(x)$ denotes the gamma function and v_0 is the thermal speed related to the kinetic temperature T_e :

$$v_0 = \sqrt{\left(\frac{2\kappa - 3}{\kappa}\right) \left(\frac{k_{\rm B} T_{\rm e}}{m_{\rm e}}\right)},\tag{3}$$

where $k_{\rm B}$ is the Boltzmann constant and $m_{\rm e}$ the electron mass.

The voltage power spectrum measured by the instrument is

$$V_{\rm r}^2 = \frac{V_{\rm e}^2 + V_{\rm p}^2 + V_{\rm s}^2}{\Gamma_{\rm r}^2},\tag{4}$$

where V_e^2 is the electron QTN using an isotropic kappa function (Le Chat *et al.*, 2009), V_p^2 is the proton thermal noise Doppler-shifted by the bulk speed of the solar wind (Issautier *et al.*, 1999; Zouganelis, 2008; Le Chat *et al.*, 2009), V_s^2 is the shot noise produced by the electrons using a kappa distribution (Le Chat *et al.*, 2010), and Γ_r^2 is the receiver gain (Chateau and Meyer-Vernet, 1991).

3. Data Selection

In situ electron measurements are performed by the low-band radio receiver of the URAP experiment onboard the *Ulysses* spacecraft (Stone *et al.*, 1992). This receiver is connected to the 2 × 35-m thin strip dipole antenna and covers the frequency range from 1.25 to 48.5 kHz in 128 s with 64 frequencies. We focus our study on the first pole-to-pole fast latitude scan between 40 and 80°S in heliocentric latitude. During this period, the high-latitude fast wind was observed in spherical expansion at constant speed between 1.5 ant 2.3 AU (Issautier *et al.*, 1998). In addition to this time selection, we add a quality criterion in order to eliminate spectra that are polluted by non-thermal emissions. The final sample contains approximately 30 000 spectra measured between September 1994 and January 1995, in a radial distance range from 1.5 to 2.3 AU. The fitting procedure was described by Le Chat *et al.* (2010): we use the proton temperature and the solar wind speed from the *Ulysses*/SWOOPS (Solar Wind Observations Over the Poles of the Sun) experiment (Bame *et al.*, 1992). We are left with three free parameters: the electron density, the total electron temperature and the kappa index, which are deduced from the fitting using 64 frequencies for each of the $\approx 30\,000$ spectra.

4. Results

4.1. Electron Density

Figure 1 shows the electron density obtained in our data set as a function of the heliocentric distance. The solid line is the best fit power law to the data. The fitted parameters are given

Figure 1 Radial variation of the electron density in the selected data set (see Section 3). The color map represents the histogram of the electron density measured by the QTN spectroscopy with the corresponding color bar chart on the right side of the figure. The solid line is the best fit power law to the data whose fitting parameters are given in Table 1.



number of cases (x 10²)

$n_{\rm e} = n_0 \times R_{\rm AU}^{-\alpha}$		$T_{\rm e} = T_0 \times R_{\rm AU}^{-\beta}$		$\kappa = \kappa_0 \times R_{\rm AU}^{-\eta}$	
α	$n_0 ({\rm cm}^{-3})$	β	$T_0(\times 10^5 \text{ K})$	η	κ
1.96 ± 0.08	2.68 ± 0.03	0.53 ± 0.15	2.3 ± 0.3	0.02 ± 0.02	2.0 ± 0.2

3

T_e (x 10⁵ K)

1

 Table 1
 Electron density, temperature and kappa index variations with heliocentric radial distance in AU obtained in the selected data set (see Section 3).

Figure 2 Radial variation of the electron temperature in the selected data set (see Section 3). The color map represents the histogram of the electron temperature measured by the QTN spectroscopy with the corresponding color bar chart on the right side of the figure. The solid line is the best fit power law to the data whose fitting parameters are given in Table 1.

in Table 1. We use the Levenberg–Marquardt algorithm to minimise the χ^2 merit function. We find that the considered data set is a good example of stationary high-speed solar wind in spherical expansion at constant speed, with no large-scale temporal or latitudinal variations in the observing time range (of about 4 months). The electron density varies as $n_e(\text{cm}^{-3}) = 2.68 \pm 0.03 \times R_{AU}^{-1.96\pm0.08}$, where R_{AU} is the heliocentric distance in astronomical units, between 1.5 to 2.3 AU, as previously found by Issautier *et al.* (1998).

1.5 1.6 1.7

1.8 1.9

R (AU)

2

2.1 2.2 2.3

4.2. Total Electron Temperature

Figure 2 shows the total electron temperature T_e , obtained by the QTN spectroscopy using a kappa function in our data set, as a function of the heliocentric distance. The solid line is the best fit of a power law ($T_e \propto R^{(\beta \pm \delta\beta)}$) to the data:

$$T_{\rm e}(\times 10^5 \,{\rm K}) = (2.3 \pm 0.3) \times R_{\rm AU}^{(-0.53 \pm 0.15)}$$
 (5)

(see Table 1 for a summary of the results). The value 0.3×10^5 K is the standard deviation of the electron temperature scaled to 1 AU using the power-law index $\beta = -0.53$. The corresponding histogram is shown in Figure 3. Since we find that the considered data set is a good example of stationary high-speed solar wind in spherical expansion between 1.5 to 2.3 AU, we can conclude that the total electron temperature in the high-speed wind can be described by the power law of Equation (5) in this range of distance.

The radial variations found, $n_e \propto R^{-1.96}$ and $T_e \propto R^{-0.53}$, suggest a polytropic relation between n_e and T_e in the selected fast solar wind, given by $T_e \propto n_e^{\gamma-1}$, with $\gamma = 1.27 \pm 0.07$. The validity of the polytropic relation is based on the absence of latitudinal and temporal variation as stated in Section 4.1. It is noteworthy that this result refers to the stationary high-speed wind at high latitude near solar activity minimum and concerns only a limited range of distance (1.5 to 2.3 AU).



4.3. Kappa Index

Figure 4 shows the kappa index obtained from our data as a function of the heliocentric distance. The solid line is the best fit power law ($\kappa \propto R^{(-\eta \pm \delta \eta)}$) to the data whose parameters are given in Table 1. We find a small value of $\kappa = 2.0 \pm 0.2$, which remains almost constant ($\eta = 0.02 \pm 0.02$) for the high-latitude fast wind in spherical expansion between 1.5 and 2.3 AU. Such a small value of κ means that the fast solar wind is highly suprathermal, since the κ index is related to the proportion of suprathermal electrons.

5. Discussion

The electron temperature found in the present study is higher (\geq 50%) than that measured by the SWOOPS electron experiment (Bame *et al.*, 1992) using the analysis procedure described in the NSSDC user's guide. There are at least three possible causes for this difference.

- i) The QTN spectroscopy is a passive method, thus any additional contribution to the power level that is not accounted for in the analysis leads to an overestimation of the QTN temperature, even though our quality criteria in the data selection (see Section 3) should considerably reduce the overestimation.
- *ii*) We consider an isotropic electron velocity distribution, but observations show that the electron velocity distribution is anisotropic (Štverák *et al.*, 2008). Since the *Ulysses*

antennas are roughly perpendicular to the magnetic field at high latitude near solar activity minimum, the quasi-thermal noise analysis yields the effective temperature $(T_{e_{\parallel}} + T_{e_{\perp}})/2$, except close to the plasma frequency peak (Meyer-Vernet, 1994). Therefore, an anisotropy of temperature such as $T_{e_{\parallel}} \approx 2 \times T_{e_{\perp}}$ (Pilipp *et al.*, 1990; Štverák *et al.*, 2008) should lead to an overestimation of the temperature of only 12%.

iii) The analysis procedure of the SWOOPS data considers distributions with an energy cut-off at 860 eV. This energy cut-off leads to an underestimation of the total electron temperature of 20% in the case of a kappa function with $\kappa = 2$. One can also consider the difficulties of electron measurements with particle detectors due to the spacecraft potential and other environmental effects (Issautier, 2009), which does not strongly affect the QTN spectroscopy.

Similarly, the 1 AU scaled total electron temperature we obtain in the fast solar wind at high latitudes ($T_e = (2.3 \pm 0.3) \times 10^5$ K) is higher than that measured at 1 AU by *Helios* (Pilipp *et al.*, 1990) and than the *Wind*/SWE data (Ogilvie *et al.*, 1995) during the same period as our data ($T_e = (1.5 \pm 0.5) \times 10^5$ K), but in the in-ecliptic fast solar wind. Contrary to the high-latitude fast solar wind, the in-ecliptic fast solar wind interacts with the slow wind. These interactions and the possible causes we raised in the previous paragraph may explain the observed difference.

The histogram of the scaled electron temperature (Figure 3) shows a log-normal behavior, with a standard deviation larger than the accuracy of our measurements. This suggests that the fluctuations observed are genuine temperature fluctuations associated with the solar wind turbulence and other variations. The application of the QTN spectroscopy with kappa function on the high resolution data of *Wind*/WAVES (Bougeret *et al.*, 1995) will allow to study this fluctuations.

The radial variation of the total electron temperature ($\beta = -0.53$) is found to lie between adiabatic and isothermal behaviors, as previously reported. We also find that the radial profile of total electron temperature is flatter than that observed for the electron core temperature in the same wind, namely $\beta_c = -0.64$ (Issautier *et al.*, 1998). This difference was previously observed for in-ecliptic solar wind (Pilipp *et al.*, 1990).

The value of κ found in the high-latitude fast solar wind is small enough to accelerate the solar wind up to 800 km s⁻¹ using exospheric models with kappa distributions (Lamy *et al.*, 2003; Zouganelis *et al.*, 2004). These models give a theoretical radial profile of the total temperature: the total electron temperature in the solar wind varies as the sum of a term $\propto R^{-4/3}$ plus a constant, with both terms of the same order of magnitude near 1 AU (Meyer-Vernet and Issautier, 1998; Meyer-Vernet *et al.*, 2003). The corresponding radial variation changes with distance with the following asymptotic behaviors: adiabatic close to the Sun and isothermal at infinity. Therefore, we compare our measurements with the expression $T_e = T_0 + T_1 R^{-4/3}$, and we find

$$T_{\rm e}(\times 10^5 {\rm K}) = (1.0 \pm 0.2) + (1.5 \pm 0.3) \times R_{\rm AU}^{-4/3}.$$
 (6)

The absolute standard deviation of this fit is comparable to that obtained with the power law (Equation (5)). Therefore, since this model and the power-law model have the same number of free parameters, one can conclude that the exospheric temperature model of the form $T_e = T_0 + T_1 R^{-4/3}$ is as good as the power-law approximation in fitting the observed total electron temperature gradient in the small radial range considered. The exospheric approach can explain the electron temperature profile in the high-latitude fast solar wind between 1.5 and 2.3 AU. In the future, the *Solar Orbiter* and *Solar Probe Plus* radio instruments will provide a large enough distance range to distinguish between a simple power-law model and the exospheric approach.

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