Quasi-thermal noise spectroscopy: preliminary comparison between kappa and sum of two Maxwellian distributions

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Abstract. Quasi-thermal noise spectroscopy has been intensively used to measure in situ the solar wind electron density and core temperature in space with various spacecraft. This method allowed study of the large-scale properties of the solar wind. This paper reminds theoretical tools to compute the quasi-thermal noise spectroscopy using a superposition of two Maxwellian distributions to describe the electrons, and the ones using a kappa distribution, which has been recently extended to non integer values of kappa. This paper presents an example of Ulysses data fitted with quasi-thermal noise using a kappa and the sum of two Maxwellians. We make a preliminary comparison of the two results.

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INTRODUCTION

By the same way as a passive electric antenna is sensitive to electromagnetic waves, it is also sensitive to local fluctuations of the electric field. These fluctuations are produced by the motions of the ambient electrons and ions. When the plasma is stable, this quasi-thermal noise (QTN) is completely determined by the particle velocity distributions in the frame of the antenna [11]. Since the shape of the spectrum is mainly determined by the electron velocity distribution, the analysis of the spectrum reveals its properties. One of the main advantages of the QTN spectroscopy is its relative immunity to the spacecraft potential and photo-electrons perturbations, which, in general, affect particle analyzers [4, 10].

In the solar wind, the mechanism of energy transport is still an open question. Measuring accurately the temperature of the electrons and their non thermal properties with quasi-thermal spectroscopy can give important clues to understand the energy transport properties. The implementation of this tool on Ulysses, assuming an electron velocity distribution made of a superposition of two Maxwellians, has given radial profiles for the electron density and core temperature with a good accuracy [3, 5, 6]. However, the radial profile of the total temperature of the electrons was less accurately determined. Analysis of the same data set, but assuming a kappa distribution, is expected to improve the accuracy of this measurement. The observations have also shown that the supra-thermal electrons are better fitted by a kappa distribution [8, 14].

In this work, we compare the QTN computed for a superposition of two Maxwellians and a kappa distribution function. We show preliminary results on data reduction with kappa distribution using in situ electron measurements performed by the low-band radio receiver of the URAP experiment on Ulysses. This receiver is connected to a $2 \times 35\,m$ thin strip dipole antenna and covers the frequency range from 1.25 to $48.5\,kHz$ in $128\,s$ [13].

QUASI-THERMAL NOISE SPECTROSCOPY

The voltage power spectrum of the plasma quasi-thermal noise at the terminals of an antenna in a plasma drifting with velocity $\vec{V}$ is

$$V_\omega^2 = \frac{2}{(2\pi)^3} \int \left| \frac{\vec{k} \cdot \vec{J}}{k} \right|^2 E^2(\vec{k}, \omega - \vec{k} \cdot \vec{V}) d^3k \qquad (1)$$

The first term in the integral involves the antenna response to electrostatic waves, which depends on the Fourier transform $\vec{J}(\vec{k})$ of the current distribution along the antenna. The second term is the auto-correlation function of the electrostatic field fluctuations in the antenna frame. At frequencies much higher than the gyrofrequency, we have

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$$E^2(\vec{k},\omega) = 2\pi \frac{\Sigma_j q_j^2 \int f_j(\vec{v})\delta(\omega - \vec{k}\cdot\vec{v})d^3v}{k^2\varepsilon_0^2 \left|\varepsilon_L(\vec{k},\omega)\right|^2} \quad (2)$$

$f_j(\vec{v})$ being the velocity distribution of the j^{th} species of charge q_j and $\varepsilon_L(\vec{k},\omega)$ the plasma longitudinal function [12].

In the case of electrons, the thermal velocity is usually higher than the plasma velocity $\vec{V}$ so, after a few manipulation using the isotropy of $f(\vec{v})$ [1, 9], we obtain

$$V_\omega^2 = \frac{16m\omega_p^2}{\pi\varepsilon_0}\int_0^\infty \frac{F(kL)B(k)}{k^2|\varepsilon_L|^2}dk \quad (3)$$

with:

$$B(k) = \frac{2\pi}{k}\int_{\omega/k}^\infty v f(v)dv \quad (4)$$

$$\varepsilon_L = 1 + \frac{2\pi\omega_p^2}{k}\int_{-\infty}^{+\infty}\frac{v_\parallel f(v_\parallel)}{kv_\parallel - \omega - io}dv_\parallel \quad (5)$$

where $v_\parallel$ is the component of $\vec{v}$ parallel to $\vec{k}$. The term io denotes an infinitesimal positive imaginary part, and the function F specifies the antenna geometry as

$$F(x) = \frac{1}{x}\left[Si(x) - \frac{1}{2}Si(2x) - \frac{2}{x}sin^4\left(\frac{x}{2}\right)\right] \quad (6)$$

for wire antenna, with Si the sine integral function.

Equations 3, 4 and 5 enable us to calculate the theoretical spectrum as a function of the velocity distribution of the particles.

Superposition of two Maxwellians

A sum of two Maxwellian functions has mainly been used for modeling the electron velocity distributions in quasi-thermal noise analysis. The first one describes the core (density n_c, temperature T_c), and the second one describes the supra-thermal halo (density n_h, temperature T_h).

In this case, the parameters obtained are: the electron density n_e, the temperature of the core distribution T_c, the ratios $\alpha = n_h/n_c$ and $\tau = T_h/T_c$.

Kappa function

We use the following generalized Lorentzian function as electron velocity distribution:

$$f_\kappa(v) = \frac{A}{(1+v^2/\kappa v_0^2)^{\kappa+1}} \quad (7)$$

with:

$$A = \frac{\Gamma(\kappa+1)}{(\pi\kappa)^{3/2}v_0^3\Gamma(\kappa-1/2)} \quad (8)$$

where $\Gamma(x)$ denotes the gamma function and v_0 is the thermal speed related to the kinetic temperature T_e as:

$$v_0 = \sqrt{\frac{2\kappa-3}{\kappa}\frac{k_BT_e}{m_e}} \quad (9)$$

where k_B is the Boltzmann constant and m_e the electron mass.

Such f_κ functions will be named in this paper ”kappa functions”; κ is a real number, which, from eq. 9 must be greater than $3/2$. In the upper limit $\kappa \to \infty$, these functions are equivalent to Maxwellians functions.

We define the Debye length in this plasma as

$$L_D = \frac{v_0}{\omega_p}\left(\frac{\kappa}{2\kappa-1}\right)^{1/2} \quad (10)$$

which is the shielding distance of low-frequency electric perturbations with a ”Kappa” distribution [2].

In the case of a kappa function as electron velocity distribution, the electron thermal noise becomes [7]:

$$\frac{V_\omega^2}{T_e^{1/2}} = \frac{16}{\pi^{3/2}\varepsilon_0\kappa}\frac{1}{r^2}m_e^{1/2}k_B^{1/2}(2\kappa-3)^{1/2}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \times \int_0^{+\infty}F\left(\frac{ru}{z(2\kappa-1)^{1/2}}\right)\frac{zdz}{|\varepsilon_L|^2(1+z^2)^\kappa} \quad (11)$$

with $z = \omega/\kappa^{1/2}kv_0$, $r = f/f_p = \omega/\omega_p$ and $u = L/L_D$, where L is the length of one antenna arm.

The parameters obtained are: the electron density n_e, the electron temperature T_e and the kappa index κ.

Shot noise and proton thermal noise

Since the antenna is a physical object, which disturbs the trajectories of the particles (they cannot pass through its surface) and furthermore the antenna surface can eject photo-electrons, there is an additional noise, which will be called shot noise in this paper.

If the antenna of radius a satisfies $a < L_D$, then a good approximation for this shot noise for a Maxwellian distribution of thermal speed $v_{the} = (2k_BT_e/m_e)^{1/2}$[9] is given by

$$V_S^2 \approx 2e^2N_e|Z|^2 \quad (12)$$

where $N_e \approx \beta n_e S < v > /4$, is the electron impact rate on one antenna arm of surface S, and $\beta = 1 + e\phi/k_BT_e$ with $\phi \approx 3.6V$ for Ulysses spacecraft. Z is the dipole antenna impedance. $<>$ denotes an average over the velocity distribution, and the term $1/4$ stems from averaging over the velocity directions of incoming particles. For a Maxwellian, we have $< v >= \sqrt{8k_BT/\pi m} =$

$2v_{the}/\sqrt{\pi}$. The electron impact rate finally becomes $N_e \approx (4\pi)^{-1/2} n_e S v_{the} \beta$.

In the case of a two Maxwellians plasma, this gives [9]:

$$N_e \approx (4\pi)^{-1/2} n_c v_{t_c} \left(\beta_c + \alpha\tau^{1/2}\right) S \tag{13}$$

with $v_{t_c} = (2k_B T_c/m_e)^{1/2}$ and $\beta_c = 1 + e\phi/k_B T_c$.

For a "kappa" plasma, $< v >$ becomes [2]:

$$< v > = 2\sqrt{\frac{\kappa}{\pi}} \frac{\Gamma(\kappa-1)}{\Gamma(\kappa-1/2)} v_0$$

with v_0 defined in the eq. 9.

This finally gives:

$$N_e \approx (4\pi)^{-1/2} n_e \beta_\kappa S \frac{\Gamma(\kappa-1)}{\Gamma(\kappa-1/2)} \sqrt{(2\kappa-3)\frac{k_B T_e}{m_e}} \tag{14}$$

with

$$\beta_\kappa = 1 + \frac{2\kappa-2}{2\kappa-3}\frac{e\phi}{k_B T_e}$$

Thus, we obtain the shot noise level by putting the eq. 13, or 14, in eq. 12, for a superposition of two Maxwellians and a kappa distribution, respectively.

In the case of the solar wind where the thermal velocity of the protons v_{thp} is smaller than the velocity of the plasma V, the lower-frequency part of the QTN spectrum is due to the above-mentioned shot noise and to the proton thermal noise, which is Doppler-shifted by the plasma velocity. This noise has been extensively studied by Issautier *et al.* [4] and applied to kappa distributions recently [7, 15].

The total noise is finally the sum of the quasi-thermal noise of the electrons, the Doppler-shifted thermal noise of the proton and the shot noise, multiplied by the receiver gain [1, 7, 9, 15]. The shot noise and the proton thermal noise only contribute at low frequency, when $f < f_p$ [4, 7, 9, 15].

Figure 1 represents two quasi-thermal noise spectra artificially separated: one for a superposition of two Maxwellians, and the other for a kappa function as electron velocity distributions. The main influences of the measured parameters are shown in the figure. Slight differences in the proton parameters do not affect the electrons parameters measured. So, in the kappa case, we use for the proton parameters T_p and V those measured by the SWOOPS experiment on Ulysses in order to save calculation time. Thus, in the next section, we fit the 64 frequencies of the URAP receiver with only three free parameters, namely n_e, T_e, and κ.

Figure 1 shows that one of the differences between this two cases is that the total electron temperature T_e is not related to the shape of the noise peak [2, 7] when using

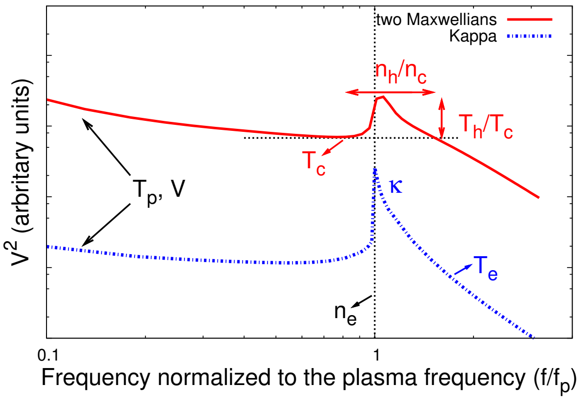

FIGURE 1. Typical QTN spectra with a kappa velocity distribution for the electrons (dash-dotted line) and a two-Maxwellians distribution (solid line). The main influences of the measured parameters are shown.

a kappa function, but is strongly related to it for the two-Maxwellians case, since:

$$T_{e_{max}} = T_c \left(\frac{1+\alpha\tau}{1+\alpha}\right) \tag{15}$$

where $\alpha = n_h/n_c$ and $\tau = T_h/T_c$. Since, for any stable distribution function, the high frequency part of the quasi-thermal noise depends only on the electron pressure [2, 7], it appears that the measured total temperature should be weakly dependant of the velocity distribution, namely $T_{e_{max}} \approx T_e$.

RESULTS

Figure 2 represents an example of power spectrum, measured in the solar wind with the URAP experiment, fitted by a theoretical quasi-thermal noise power spectrum with a sum of two Maxwellians and a kappa electron velocity distributions. The best fitted electron parameters for the two Maxwellians are: $n_e = 2.79 \pm 0.04\, cm^{-3}$, $T_c = 7.1 \pm 0.4 \cdot 10^4\, K$, $\alpha = 0.04 \pm 0.01$ and $\tau = 8.5 \pm 0.9$. With the kappa distribution, the fitted parameters are: $n_e = 2.83 \pm 0.01\, cm^{-3}$, $T_e = 9.0 \pm 0.2 \cdot 10^4\, K$, and $\kappa = 3.1 \pm 0.1$.

The total temperature for the superposition of two Maxwellians, obtained using eq. 15, is: $T_{e_{max}} = 9.2 \pm 1.2 \cdot 10^4\, K$, which is in good agreement with the temperature obtained with the kappa distribution. The accuracy of the total electron temperature is better using a kappa distribution than using a sum of two Maxwellians. Note that the URAP receiver gives each spectrum every $128\, s$, in which time the plasma properties may change. Moreover, the accuracy of the electron density depends on the

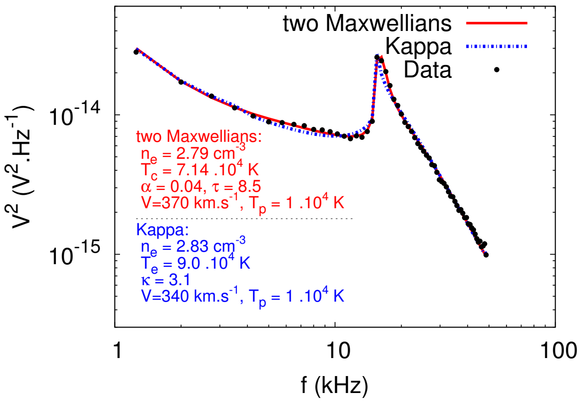

FIGURE 2. Example of power spectrum (in $V^2 Hz^{-1}$) measured in the solar wind with the URAP receiver connected to the $2 \times 35\,m$ dipole antenna, at about 1.3 AU from the Sun and 6°N of heliocentric latitude. The data are plotted as heavy dots. The solid line shows the theoretical QTN for two Maxwellians electron distribution, which corresponds to the best fit of the data. The dash-dotted line is the theoretical QTN for kappa electron distribution, which corresponds to the best fit of the data. The best fit parameters are shown.

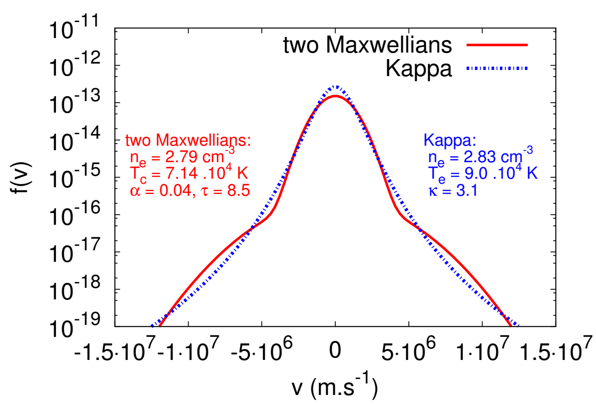

FIGURE 3. Comparison of the sum of two Maxwellians and the kappa electron velocity distribution compute with the best fit electron parameters of the figure 2. The corresponding parameters are shown.

frequency resolution, which is $750\,Hz$ for this data. Both effects tend to round up the measured peak at the plasma frequency.

Figure 3 compares the electron velocity distributions corresponding to the best fits of the figure 2.

DISCUSSION

Preliminary results of quasi-thermal noise analysis using kappa electron velocity distribution are promising. In particular, the density and temperature measurements agree with the ones obtained with a superposition of two Maxwellians. Furthermore, the accuracy of the total electron temperature in the kappa case is improved, compared to the one estimated by a sum of two Maxwellians, although the fitting using a kappa distribution uses less free parameters. Detailed analysis of the comparison between a kappa function and a sum of two Maxwellians needs further studies. We will apply the QTN method using a kappa function to the Ulysses data in routine, in particular during its three fast polar passes to determine accurately the radial variation of the total electron temperature.

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